Bitwise Operators

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Spring 2023

Bitwise Operation

- Logical operator
  - Bitwise complement: ~
  - Bitwise AND: &
  - Bitwise inclusive OR: |
  - Bitwise exclusive OR: ^

```
X = 4190: 0001 0000 0101 1110
Y = 103:  0000 0000 0110 0111

X & Y:    0000 0000 0100 0110
X | Y:    0001 0000 0111 1111
X ^ Y:    0001 0000 0011 1001
```

- Shift operator
  - Left shift: <<
  - Right shift: >>

NOT	
X	~X
0	1
1	0

AND		
X	Y	X & Y
0	0	0
0	1	0
1	0	0
1	1	1

OR		
X	Y	X | Y
0	0	0
0	1	1
1	0	1
1	1	1

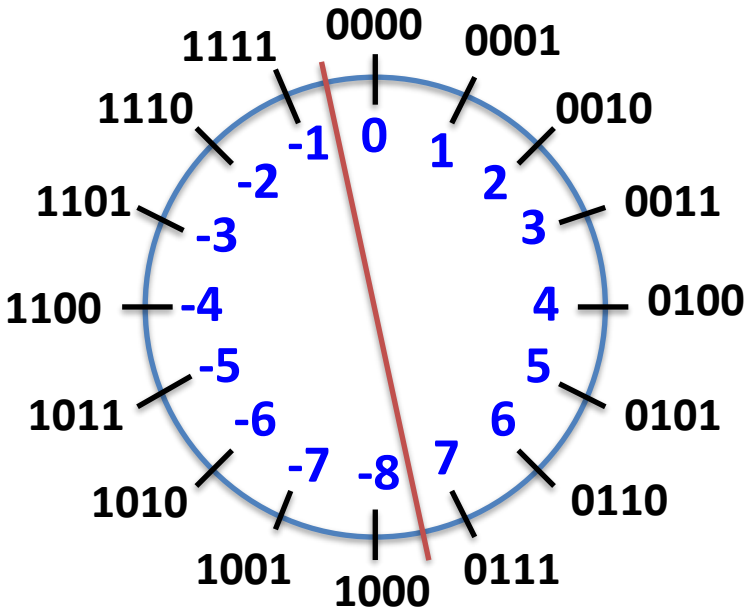
XOR		
X	Y	X ^ Y
0	0	0
0	1	1
1	0	1
1	1	0

Operator Precedence & Associativity

Operator	Associativity
`()` `[]` `++` (*postfix*) `--` (*postfix*)	Left to right
`+` (*unary*) `-` (*unary*) `++` (*prefix*) `--` (*prefix*) `!` `&` (*address*) `*` (*dereference*) `~`	Right to left
`*` `/` `%`	Left to right
`+` `-`	Left to right
`<<` `>>`	Left to right
`<` `<=` `>` `>=`	Left to right
`==` `!=`	Left to right
`&`	Left to right
`^`	Left to right
`|`	Left to right
`&&`	Left to right
`||`	Left to right
`?:`	Right to left
`=` `+=` `-=` `*=` `/=` `%=` `<<=` `>>=` `&=` `^=` `|=`	Right to left
`,` (*comma operator*)	Left to right

Two's Complement (1)

- Unique zero
- Easy for hardware
  - leading 0 ≥ 0
  - leading 1 < 0
- Used by almost all modern machines



$$O(B) = -b_{w-1} \cdot 2^{w-1} + \left(\sum_{i=0}^{w-2} b_i \cdot 2^i \right)$$

Asymmetric range:

$$-2^{n-1} \sim 2^{n-1} - 1$$

Two's Complement (2)

- For *-n*, take the bitwise complement of *n* and add 1 to it:
  - **`~n + n == 1111...11`$_2$ `== -1` ➔ `~n + 1 == -n`**

```
short int n;
```

Value of n	Binary representation	Binary complement	Two's complement of -n	Value of -n
1	00000000 00000001	11111111 11111110	11111111 11111111	-1
7	00000000 00000111	11111111 11111000	11111111 11111001	-7
8	00000000 00001000	11111111 11110111	11111111 11111000	-8
9	00000000 00001001	11111111 11110110	11111111 11110111	-9
-9	11111111 11110111	00000000 00001000	00000000 00001001	9
0	00000000 00000000	11111111 11111111	00000000 00000000	0

Bitwise Binary Operators

Declarations and initializations

```
int a = 33333, b = -77777;
```

Expression	Binary representation	Value
a	00000000 00000000 10000010 00110101	33333
b	11111111 11111110 11010000 00101111	-77777
a & b	00000000 00000000 10000000 00100101	32085
a ^ b	11111111 11111110 01010010 00011010	-110054
a | b	11111111 11111110 11010010 00111111	-77249
~(a | b)	00000000 00000001 00101101 11000000	77248

Some "Boolean" Properties

- a & 0 == 0
- a & (-1) == a
- a & a == a
- a & (~a) == 0
- ~(~a) == a
- a & b == b & a
- a | b == b | a
- a ^ b == b ^ a
- a & (b | c) == (a & b) | (a & c)
- ~(a | b) == ~a & ~b

- a | 0 == a
- a | (-1) == -1
- a | a == a
- a | (~a) == -1

- a ^ 0 == a
- a ^ (-1) = ~a
- a ^ a == 0
- a ^ (~a) == -1

- a & (b & c) == (a & b) & c
- a | (b | c) == (a | b) | c
- a ^ (b ^ c) == (a ^ b) ^ c
- a | (b & c) == (a | b) & (a | c)
- ~(a & b) == ~a | ~b

Example: Bufferless Swap

- Note: `(a ^ b) ^ b == a ^ (b ^ b) == a`

```c
#include <stdio.h>

int main(void)
{
    int a = 3, b = 9;          /* a <- 0011, b <- 1001 */

    printf("a = %d, b = %d\n", a, b);

    a = a ^ b;                 /* a <- 0011 ^ 1001 = 1010 */
    b = a ^ b;                 /* b <- 1010 ^ 1001 = 0011 */
    a = a ^ b;                 /* a <- 1010 ^ 0011 = 1001 */
                               /* also try: a ^= b ^= a ^= b */
    printf("a = %d, b = %d\n", a, b);
    return 0;
}
```

An Interview Question

- Given a set of numbers where all elements occur even a number of times except one number, find the odd occuring number
  - `findodd(12, 36, 7, 12, 9, 36, 7, 12, 12)` → 9

```c
#include <stdio.h>

int findodd(int a[], int n)
{
    int i, res = 0;

    for (i = 0; i < n; i++)
        res ^= a[i];
    return res;
}
```

```c
int main(void)
{
    int odd;
    int a[] = {12, 36, 7, 12, 9, 36, 7, 12, 12};

    odd = findodd(a, sizeof(a)/sizeof(int));
    printf("%d\n", odd);
    return 0;
}
```

Shift Left Operator

- *expr1* << *expr2*
  - Shift left *expr1* by *expr2* bits (remaining bits on the right side are filled with *zeroes*)

Declarations and initializations		
`int c = 90, d = -12;`		
Expression	**Binary representation**	**Action**
`c`	`00000000 00000000 00000000 01011010`	**unshifted**
`c << 1`	`00000000 00000000 00000000 10110100`	**left-shifted 1**
`c << 4`	`00000000 00000000 00000101 10100000`	**left-shifted 4**
`c << 31`	`00000000 00000000 00000000 00000000`	**left-shifted 31**
`d`	`11111111 11111111 11111111 11110100`	**unshifted**
`d << 28`	`01000000 00000000 00000000 00000000`	**left-shifted 28**

Logical Shift Right Operator

- *expr1* >> *expr2* (when *expr1* is an unsigned integer)
  - Shift right *expr1* by *expr2* bits (remaining bits on the left side are filled with *zeroes*)

Declarations and initializations		
`unsigned int u = 90, v = 0xdeadbeef;`		
Expression	**Binary representation**	**Action**
u	00000000 00000000 00000000 01011010	unshifted
u >> 1	00000000 00000000 00000000 00101101	right-shifted 1
u >> 4	00000000 00000000 00000000 00000101	right-shifted 4
v	11011110 10101101 10111110 11101111	unshifted
v >> 8	00000000 11011110 10101101 10111110	right-shifted 8
v >> 24	00000000 00000000 00000000 11011110	right-shifted 24

Arithmetic Shift Right Operator

- *expr1* >> *expr2* (when *expr1* is a signed integer)
  - Shift right *expr1* by *expr2* bits (remaining bits on the left side are filled with *sign bits*)

Declarations and initializations

```
int m = 90, n = 0xdeadbeef;
```

Expression	Binary representation	Action
`m`	`00000000 00000000 00000000 01011010`	unshifted
`m >> 1`	`00000000 00000000 00000000 00101101`	right-shifted 1
`m >> 4`	`00000000 00000000 00000000 00000101`	right-shifted 4
`n`	`11011110 10101101 10111110 11101111`	unshifted
`n >> 8`	`11111111 11011110 10101101 10111110`	right-shifted 8
`n >> 24`	`11111111 11111111 11111111 11011110`	right-shifted 24

A Mask

- A constant or variable that is used to extract desired bits from another variable or expression
- Examples
  - The constant value 1:
    used to determine the low-order bit of an `int` expression
  - The constant value 255:
    a mask for the low-order byte

```c
/* For n >= 0,
   returns 1 if n is an odd number */
int odd(int n)
{
    int mask = 1;
    return (n & mask);
}
```

```c
/* For n >= 0, returns n % 256 */

int mod256(int n)
{
    int mask = 255;
    return (n & mask);
}
```

Example: Printing an Integer Bitwise

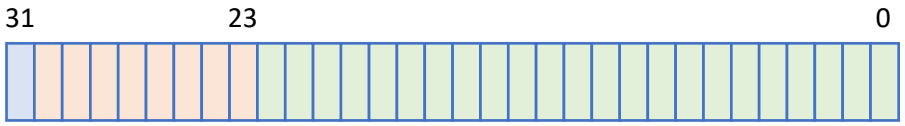
```c
#include <stdio.h>
#define BITS_PER_BYTE   8

void bit_print(int a)
{
    int i;
    int n = sizeof(int) * BITS_PER_BYTE;
    int mask = 1 << (n-1);

    for (i = 1; i <= n; i++)
    {
        putchar(((a & mask)? '1' : '0'));
        a <<= 1;         /* a = a << 1; */
        if (i % BITS_PER_BYTE == 0 && i < n)
            putchar(' ');
    }
}
```

Example: Extracting Bits from an Integer

- For a given integer,
  - Extract bit 31
  - Extract bits 30 ~ 23
  - Extract bits 22 ~ 0




```c
#include <stdio.h>

int bit31(int n) {
    return (n >> 31) & 1;
}

int bit30_23(int n) {
    return (n >> 23) & 0xff;
}

int bit22_0(int n) {
    return n & 0x7fffff;
}
```

Enumeration Types (1)

- User defined type
  - Provides a means of naming a finite set of integers and of declaring identifiers of the set
  - `enum day {Sun, Mon, Tue, Wed, Thu, Fri, Sat};` // 0, 1, 2, 3, 4, 5, 6
  - `enum fruit {apple, pear, orange, lemon};` // 0, 1, 2, 3
  - `enum fruit {apple=7, pear, orange, lemon};` // 7, 8, 9, 10

- Variable declaration
  - ```
    enum day d1, d2
    d1 = Fri;
    if (d2 == Thu) ...          /* if (d2 == 4) ... */
    ```
    ```

Enumeration Types (2)

```c
#include <stdio.h>
enum day {Sun, Mon, Tue, Wed, Thu, Fri, Sat};

enum day find_next_day(enum day d) {
    switch (d) {
        case Sun: return Mon;
        case Mon: return Tue;
        case Tue: return Wed;
        case Wed: return Thu;
        case Thu: return Fri;
        case Fri: return Sat;
        case Sat: return Sun;
    }
}
int main(void) {
    enum day d;
    for (d = Sun; d <= Sat; d++)
        printf("next day of %d is %d\n", d, find_next_day(d));
    return 0;
}
```

```c
enum day find_next_day2(enum day d) {
    return ((d + 1) % (Sat - Sun + 1));
}
```